partial drying of the structure; Δp_{fl}, viscous pressure losses in the fluid; $N = \sigma r/\nu_{fl}$, characteristic parameter of the heat carrier; σ, coefficient of surface tension of the fluid; r is the latent heat of vaporization; ν_{fl} and ν_v, kinematic coefficients of viscosity of the fluid and of the vapor on the saturation curve.

LITERATURE CITED

1. M. G. Semena and A. N. Gershuni, "Investigation of the maximum heat transport capacity of heat pipes with metal fiber wicks," Teplofiz. Vys. Temp., 16, No. 5, 1060-1067 (1978).
2. M. G. Semena, A. N. Gershuni, and B. M. Rassamakin, "Analytic investigation of the maximum heat transport capacity of heat pipes," Izv. Vyssh. Uchebn. Zaved., Energetika, No. 5, 93-97 (1977).
3. M. G. Semena and A. N. Gershuni, "Analysis of capillary—transport characteristics of metal fiber structures," Inzh.-Fiz. Zh., 41, No. 1, 5-12 (1981).

NONSTATIONARY DOPPLER EFFECT FOR WAVES PROPAGATING IN NONHOMOGENEOUS MEDIA

V. I. Krylovich

UDC 536.2:534.6

The theory of the frequency shift of continuous waves propagating in a medium whose properties depend on the coordinates and time is presented.

The fundamentals of the nonstationary frequency shift theory for waves propagating in a homogeneous medium whose properties vary in time are presented in [1-3]; the merits of the measurement and inspection methods based on this effect are demonstrated. In the general case, the medium under investigation can be nonhomogeneous, i.e., the wave propagation velocity in this medium not only can vary in time as a certain process develops, but it may also depend on the coordinates. A multilayer system provides a typical example.

If one assumes that the wave propagation velocity in the medium depends on all of the space coordinates and the time, i.e., $v = v(x, y, z; \tau)$, consideration of the frequency shift problem is made very difficult by the accompanying phenomena of phase surface distortion, interference, and diffraction. Therefore, we shall introduce certain simplifications which will make it possible to separate the above effect in "pure form." Consider the unidimensional case of propagation of a plane wave along the x axis in a medium whose properties depend only on the single coordinate x and the time τ, i.e., $v = v(x, \tau)$. Moreover, we neglect wave reverberations between the different layers of the medium.

The essence of the method used in [1-3] for determining the frequency shift consists in utilizing the law of motion of the phase surface of waves with a certain phase φ in the medium. The integral relationships given in these papers account for the change in the wave propagation velocity as the wave passes through the investigated section of the medium. It can readily be shown that they can be derived by solving the differential equation

$$\frac{dx}{d\tau} = v(\tau), \tag{1}$$

where $v(\tau)$ is the time dependence of the wave velocity in the medium. Actually, the general solution of Eq. (1) has the following form:

$$x(\tau) = \int v(\tau)\, d\tau + C = S(\tau) + C, \tag{2}$$

where $S(\tau)$ is the primitive function of $v(\tau)$, and C is the integration constant. If a wave with the phase φ is located at the point x_1 at the instant of time τ_1 (i.e., if it has entered the section of the medium under investigation), we have $x_1 = S(\tau_1) + C$, whence $C = x_1 - S(\tau_1)$, and we thus obtain the particular solution of Eq. (1),

---

Institute of Applied Physics, Academy of Sciences of the Belorussian SSSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 43, No. 4, pp. 609-615, October, 1982. Original article submitted December 10, 1981.

0022-0841/82/4304-1120$07.50

$$x(\tau) = x_1 + S(\tau) - S(\tau_1), \tag{3}$$

which represents the law of motion of a wave that has entered the investigated section of the medium at the time τ_1. If this wave is located at the point x_2 at the instant of time τ_2 (i.e., if it has left the investigated section), we write $x_2 = x_1 + S(\tau_2) - S(\tau_1)$, or

$$x_2 - x_1 \equiv L = S(\tau_2) - S(\tau_1) = \int_{\tau_1}^{\tau_2} v(\tau)\,d\tau, \tag{4}$$

which coincides with expression (1) in [2]. Similarly, we can derive the other equations given in [2].

In order to establish the relationship between the frequencies of the same wave at the times of its passage across the boundaries of the investigated section of the medium, we write (2) in the following form:

$$x(\tau) - S(\tau) - C \equiv F(x,\ \tau;\ C) = 0, \tag{5}$$

and we write for the particular solution (3)

$$F(x,\ \tau;\ x_1,\ \tau_1) = 0, \tag{6}$$

and, instead of relationship (4),

$$F(x_2,\ \tau_2;\ x_1,\ \tau_1) = 0. \tag{7}$$

For a wave with the phase $\varphi + \Delta\varphi$ we similarly obtain

$$F(x_2,\ \tau_2 + \Delta\tau_2;\ x_1,\ \tau_1 + \Delta\tau_1) = 0. \tag{8}$$

Using the latter expression to define the relationship

$$\lim_{\Delta\varphi \to 0} \frac{\Delta\tau_2}{\Delta\tau_1} = \frac{d\tau_2}{d\tau_1} = -\frac{\partial F/\partial\tau_1}{\partial F/\partial\tau_2} \tag{9}$$

and considering that (see Eq. (5) in [3])

$$\lim_{\Delta\varphi \to 0} \frac{\Delta\tau_2}{\Delta\tau_1} = \frac{f_1(\tau_1)}{f_2(\tau_2)},$$

we finally obtain

$$f_2(\tau_2) = -f_1(\tau_1)\,\frac{\partial F(x_2,\ \tau_2;\ x_1,\ \tau_1)/\partial\tau_2}{\partial F(x_2,\ \tau_2;\ x_1,\ \tau_1)/\partial\tau_1}. \tag{10}$$

Relationship (10) has a sufficiently high degree of generality; it relates the frequency of the wave at the time of its reception to the frequency of the same wave at the moment of emission (if the emitter and the receiver are located at the boundaries of the investigated section of the medium). If $f_1(\tau) = f_0 = \text{const}$ and the wave source and receiver are immobile (x_2 and x_1 are independent of time), we obtain from the above relationship the equation for the nonstationary Doppler effect; assuming that $x_1 = x_1(\tau)$ and $x_2 = x_2(\tau)$, we obtain from (10) directly the generalized relationship given in [3].

The approach presented above can also be used for an inhomogeneous medium. The only difference is that, for the given constraints, we seek in this case, instead of Eq. (1), the solution of the differential equation

$$\frac{dx}{d\tau} = v(x,\ \tau) \tag{11}$$

for the initial condition

$$x(\tau_1) = x_1. \tag{12}$$

The measurement and inspection methods based on the nonstationary frequency shift consist in comparing continuously during an experiment the received and the emitted signals and separating and recording their frequency difference, in which a harmonic signal with highly stable frequency, i.e., $f_1(\tau) = f_0 = \text{const}$, is supplied to the emitter. In this case, the wave reception time τ_2 can be conveniently considered as the present time $\tau_2 = \tau$, while $\tau_1 =$

$\tau - \tau_3$, where τ_3 is the time during which a wave recorded at the given instant of time τ traverses the investigated section of the medium, i.e., it is the lag time. Considering this, we write the following instead of (7):

$$F(x_2, \tau; x_1, \tau - \tau_3) = 0. \tag{13}$$

Relationship (13) is the equation for determining τ_3 for the assigned values of x_1, x_2, and τ. Comparing (7) and (13), we obtain

$$\frac{d\tau_2}{d\tau_1} = \frac{d\tau}{d(\tau - \tau_3)} = \frac{1}{1 - \dfrac{d\tau_3}{d\tau}}. \tag{14}$$

Taking into account (13) and assuming that $f_1(\tau) = f_0 = \text{const}$, we obtain instead of (10) the more convenient expression

$$\Delta f(\tau) = -f_0 \frac{d\tau_3}{d\tau} = f_0 \frac{\partial F/\partial \tau}{\partial F/\partial \tau_3}, \tag{15}$$

where $\Delta f(\tau) = f_2(\tau) - f_0$; F is determined by expression (13) and is found as the implicit solution of Eq. (11), which satisfies the conditions

$$x(\tau - \tau_3) = x_1, \quad x(\tau) = x_2. \tag{16}$$

It should be mentioned that it is usually impossible to derive an explicit analytical dependence of τ_3 on τ from (13) [4]. Therefore, in spite of the apparent simplicity of the theory presented above, calculation of the relationship $\Delta f(\tau)$ for specific experimental conditions involves great difficulties and usually requires the use of numerical methods and computers. However, in many cases of practical importance, the magnitude of the frequency shift is negligibly small in comparison with the frequency of emitted waves, i.e., $\Delta f \ll f_0$. Using this condition, we can derive an approximate expression for the frequency shift involving a relative error of the order of $\Delta f/f_0$ in comparison with the exact expression by utilizing the following relationship as the initial expression [1, 2]

$$\Delta f = f_0 \frac{L}{v^2} \frac{dv}{d\tau}, \tag{17}$$

which holds for a homogeneous medium.

Assume that an explicit dependence of the wave velocity on the coordinate x and the time τ, $v = v(x, \tau)$, is assigned in the investigated interval [0, L] of the medium between the wave source and the receiver. We subdivide the interval [0, L] into n segments with the boundary coordinates 0, x_1, x_2, ..., x_{n-1}, $x_n = L$; the lengths of the thus obtained segments are correspondingly equal to $\Delta x_1 = x_1$, $\Delta x_2 = x_2 - x_1$, $\Delta x_3 = x_3 - x_2$, ..., $\Delta x_n = L - x_{n-1}$. We consider that the wave propagation velocity within each segment is independent of the coordinate, assuming that, for each segment, $v_1(\tau) = v(x_1, \tau)$, $v_2(\tau) = v(x_2, \tau)$, ..., $v_n(\tau) = v(x_n, \tau)$, respectively. We write successively the approximate expressions for the frequency shift acquired by a wave emitted at the instant of time τ_0 with the frequency $f_0 = \text{const}$ within each segment:

$$\left.\begin{aligned}
&\Delta f_1 \approx f_0 \frac{\Delta x_1}{v_1^2(\tau_0)} \frac{dv_1(\tau_0)}{d\tau}, \\
&\Delta f_2 \approx (f_0 + \Delta f_1) \frac{\Delta x_2}{v_2^2(\tau_1)} \frac{dv_2(\tau_1)}{d\tau}, \quad \tau_1 = \tau_0 + \frac{\Delta x_1}{v_1(\tau_0)}, \\
&\Delta f_3 \approx (f_0 + \Delta f_1 + \Delta f_2) \frac{\Delta x_3}{v_3^2(\tau_2)} \frac{dv_3(\tau_2)}{d\tau}, \quad \tau_2 = \tau_0 + \\
&\qquad + \frac{\Delta x_1}{v_1(\tau_0)} + \frac{\Delta x_2}{v_2(\tau_1)}, \\
&\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
&\Delta f_n \approx \left(f_0 + \sum_{i=1}^{n-1} \Delta f_i\right) \frac{\Delta x_n}{v_n^2(\tau_{n-1})} \frac{dv_n(\tau_{n-1})}{d\tau}, \quad \tau_{n-1} =
\end{aligned}\right\} \tag{18}$$

$$= \tau_0 + \sum_{i=1}^{n=1} \frac{\Delta x_i}{v_i(\tau_{i-1})}.$$

The resulting change in the oscillation frequency arising as the wave traverses the interval [0, L], is obviously equal to the algebraic sum of the frequency shifts developing successively within each segment:

$$\Delta f(\tau) = \sum_{k=1}^{n} \Delta f_k(\tau). \tag{19}$$

We rewrite expression (18) in a different form:

$$\left.\begin{aligned} \Delta f_1 &\approx f_0\alpha_1, \\ \Delta f_2 &\approx f_0\alpha_2 + \Delta f_1\alpha_2, \\ \Delta f_3 &\approx f_0\alpha_3 + \Delta f_1\alpha_3 + \Delta f_2\alpha_3, \\ \Delta f_4 &\approx f_0\alpha_4 + \Delta f_1\alpha_4 + \Delta f_2\alpha_4 + \Delta f_3\alpha_4, \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ \Delta f_n &\approx f_0\alpha_n + \Delta f_1\alpha_n + \Delta f_2\alpha_n + \ldots + \Delta f_{n-1}\alpha_n, \end{aligned}\right\} \tag{20}$$

where

$$\alpha_i = \frac{\Delta x_i}{v_i^2(\tau_{i-1})} \frac{dv_i(\tau_{i-1})}{d\tau}, \quad i = 1, 2, \ldots, n. \tag{21}$$

Adding expressions (20), we obtain

$$\sum_{i=1}^{n} \Delta f_i = \Delta f \approx f_0 \sum_{i=1}^{n} \alpha_i + \Delta f_1 \sum_{i=2}^{n} \alpha_i + \Delta f_2 \sum_{i=3}^{n} \alpha_i + \Delta f_3 \sum_{i=4}^{n} \alpha_i + \ldots + \Delta f_n\alpha_n. \tag{22}$$

For $\Delta x_i \to 0$, $n \to \infty$, the right-hand side of relationship (22) evidently approaches the actual value $\Delta f(\tau)$. We shall provide an estimate of this series. Assume that the derivative $dv_i(\tau_{i-1})/d\tau$ has the same sign within all segments; in this case, all Δf_i will have the same sign, and the sum of the series, i.e., Δf, will be at a maximum. If this condition is not satisfied, some terms of the series will have different signs, and the sum will diminish. Assuming for the sake of determinacy that $\alpha_i > 0$ ($i = 1, 2, \ldots, n$), we arrive at the obvious inequality

$$\sum_{i=1}^{n} \alpha_i > \sum_{i=2}^{n} \alpha_i > \sum_{i=3}^{n} \alpha_i > \ldots > \alpha_n.$$

Considering this, we substitute the first sum for all the sums on the right-hand side of Eq. (22), and then obviously,

$$f_0 \sum_{i=1}^{n} \alpha_i + \Delta f_1 \sum_{i=2}^{n} \alpha_i + \Delta f_2 \sum_{i=3}^{n} \alpha_i + \ldots + \Delta f_n\alpha_n < f_0 \sum_{i=1}^{n} \alpha_i +$$

$$+ (\Delta f_1 + \Delta f_2 + \ldots \Delta f_n) \sum_{i=1}^{n} \alpha_i = (f_0 + \Delta f) \sum_{i=1}^{n} \alpha_i = f_0 \left(1 + \frac{\Delta f}{f_0}\right) \sum_{i=1}^{n} \alpha_i,$$

so that the following inequality holds:

$$f_0 \sum_{i=1}^{n} \alpha_i < \Delta f < f_0 \left(1 + \frac{\Delta f}{f_0}\right) \sum_{i=1}^{n} \alpha_i. \tag{23}$$

It is evident from (23) that, with a relative error not exceeding $\Delta f/f_0$, we can neglect on the right-hand side of relationship (22) all terms of the second and higher orders, thus obtaining

$$\Delta f \approx f_0 \sum_{i=1}^{n} \frac{\Delta x_i}{v_i^2(\tau_{i-1})} \frac{dv_i(\tau_{i-1})}{d\tau}. \tag{24}$$

We also take into account the fact that, in actual cases in practice, the following condition is always satisfied:

$$\frac{dv}{d\tau} \tau_3 \ll v. \tag{25}$$

Violation of inequality (25) in a laboratory experiment can occur only in the case of fast processes of the explosion type. We shall illustrate this by using the following example. Assume that we are investigating a certain process in water by means of continuous ultrasound waves, using the nonstationary Doppler effect; the distance between the ultrasound emitter and receiver in water is equal to L = 1 m, and the velocity of ultrasound is v = 1500 m/sec; then, $\tau_3 = L/v = 1/1500$ (sec). Condition (25) is violated if $dv/d\tau \sim v/\tau_3 = 2.25 \cdot 10^6$ m/sec^2. It is difficult even to imagine a process which would produce such a rapid change in the ultrasound velocity. Therefore, with a negligible error, we can put

$$v_i(\tau_{i-1}) = v_i(\tau_0), \quad \frac{dv_i(\tau_{i-1})}{d\tau} = \frac{dv_i(\tau_0)}{d\tau},$$

while τ_0 can be considered as the present time, measured, for instance, from the start of measurements. Taking this into account, we obtain the following instead of relationship (24):

$$\Delta f(\tau) \approx f_0 \sum_{i=1}^{n} \frac{\Delta x_i}{v_i^2(\tau)} \frac{dv_i(\tau)}{d\tau} \equiv f_0 \sum_{i=1}^{n} \frac{\Delta x_i}{v^2(x_i, \tau)} \frac{dv(x_i, \tau)}{d\tau}.$$

Considering that

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} \frac{\Delta x_i}{v^2(x_i, \tau)} \frac{dv(x_i, \tau)}{d\tau} = \int_0^L \frac{dx}{v^2(x, \tau)} \frac{dv(x, \tau)}{d\tau},$$

we obtain the following expression for the nonstationary frequency shift of waves propagating in an inhomogeneous medium:

$$\Delta f(\tau) = f_0 \int_0^L \frac{dx}{v^2(x, \tau)} \frac{dv(x, \tau)}{d\tau}. \tag{26}$$

Expression (26) can be represented in a different form if the variables x and τ are considered to be independent. Then,

$$\frac{dv(x, \tau)}{d\tau} \frac{dx}{v^2(x, \tau)} = -\frac{d}{d\tau}\left(\frac{dx}{v(x, \tau)}\right)$$

holds, and

$$\Delta f(\tau) = -f_0 \frac{d}{d\tau} \int_0^L \frac{dx}{v(x, \tau)}. \tag{27}$$

The quantity

$$\int_0^L \frac{dx}{v(x, \tau)} = \tau_3(\tau) \tag{28}$$

has the meaning of the time during which the signal (wave) traverses the path interval [0, L], calculated for the instant of time τ, i.e., it has the meaning of the instantaneous value of the lag time. In this case, the relationship $L/\tau_3(\tau) = \bar{v}(\tau)$ provides the instantaneous value of the velocity averaged with respect to the coordinate at the instant of time τ. Considering this, we obtain

$$\Delta f(\tau) = -f_0 \frac{d}{d\tau}\left[\frac{L}{\bar{v}(\tau)}\right], \qquad (29)$$

or, for L = const

$$\Delta f(\tau) = f_0 \frac{L}{\bar{v}^2(\tau)} \frac{d\bar{v}(\tau)}{d\tau}. \qquad (30)$$

Thus, we have arrived at an expression similar to (17), the only difference being that $\bar{v}$ refers to the instantaneous value of the wave propagation velocity, averaged over a section of the medium.

NOTATION

v, wave propagation velocity in the medium; x, y, z, coordinates; τ, time; τ_3, time during which the wave traverses the interval of the medium under investigation; L, length of interval; f_1 and f_0, frequencies of the waves emitted into the medium; f_2, frequency of the waves that have passed through the medium.

LITERATURE CITED

1. N. V. Antonishin and V. I. Krylovich, "Prospects for the use of the acoustic Doppler effect in thermophysics and other fields," in: Heat and Mass Transport [in Russian], Vol. 7, Énergiya, Moscow (1966).
2. V. I. Krylovich, "Nonstationary Doppler effect and frequency—phase methods of investigation and inspection," Inzh.-Fiz. Zh., 36, No. 3, 487-492 (1979).
3. V. I. Krylovich, "Effect of the shift in the frequency of received waves," Inzh.-Fiz. Zh., 41, No. 3, 507-513 (1981).
4. G. A. Korn and T. M. Korn, Manual of Mathematics, McGraw-Hill (1967).